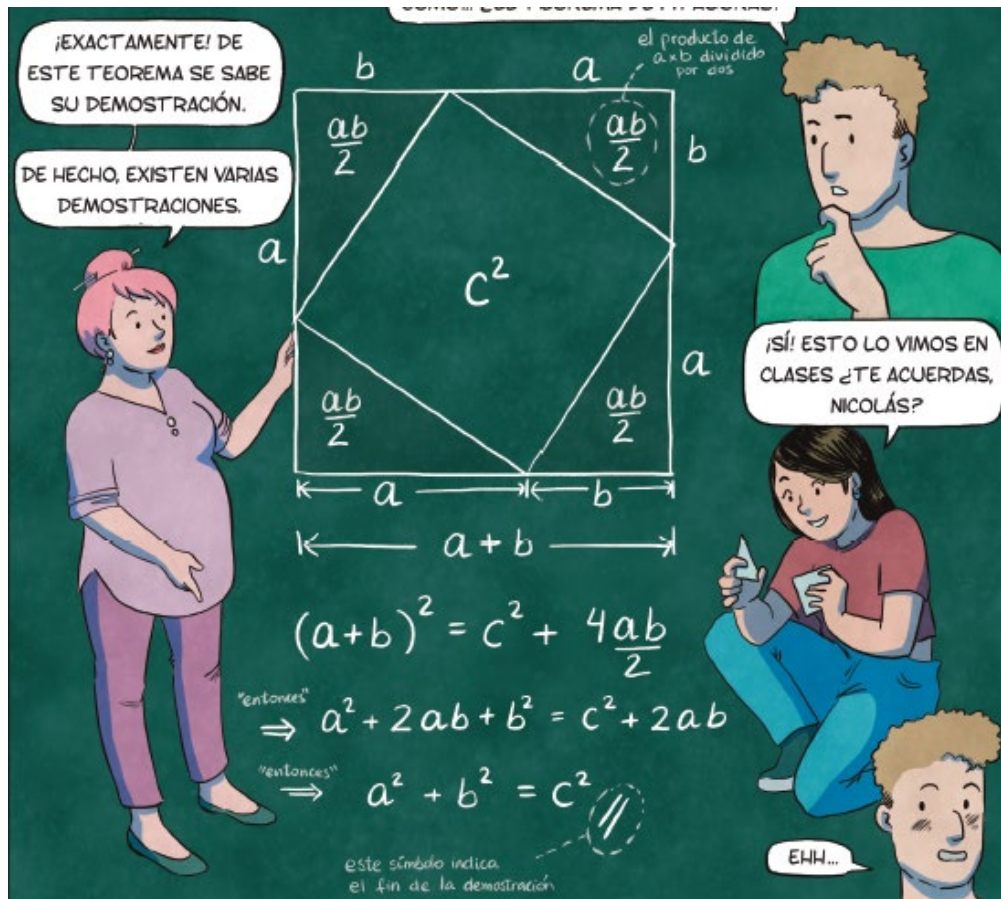


Visualizing mathematics with sketchnotes and comics

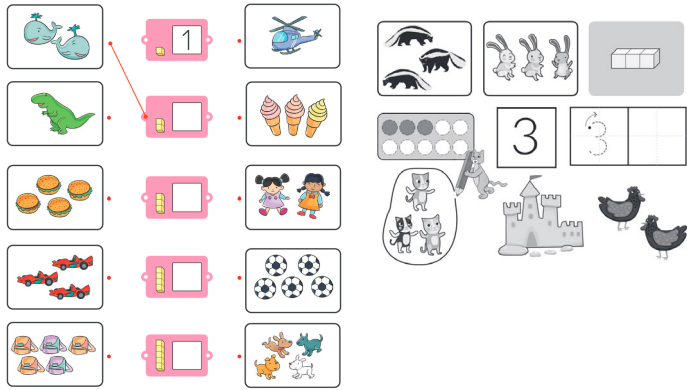


Constanza Rojas-Molina
CY Cergy Paris Université
www.crojasmolina.com

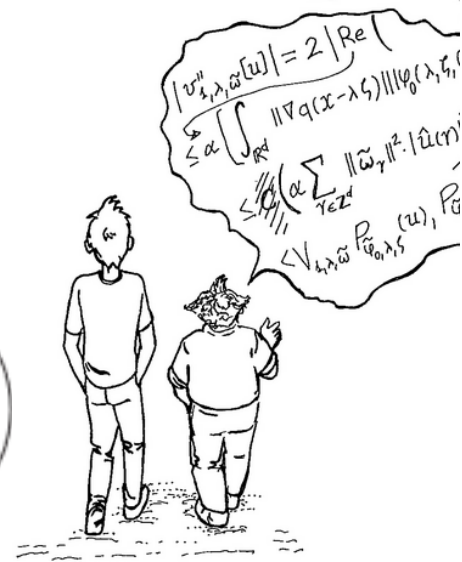
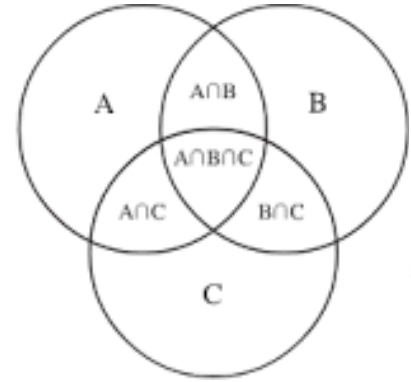
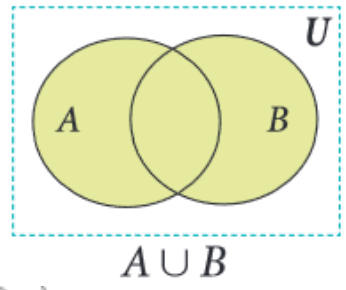
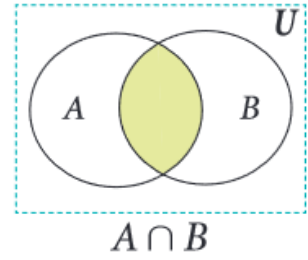
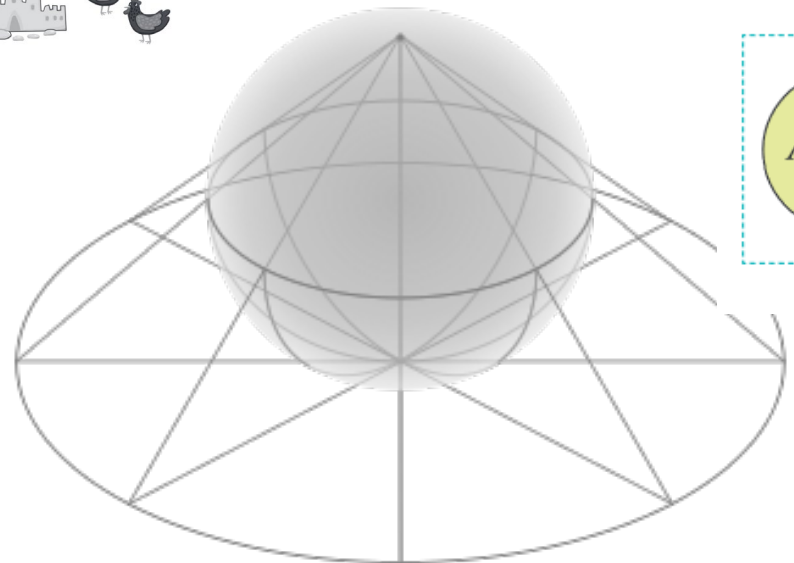


@e.a.casanova

Maths: a visual language



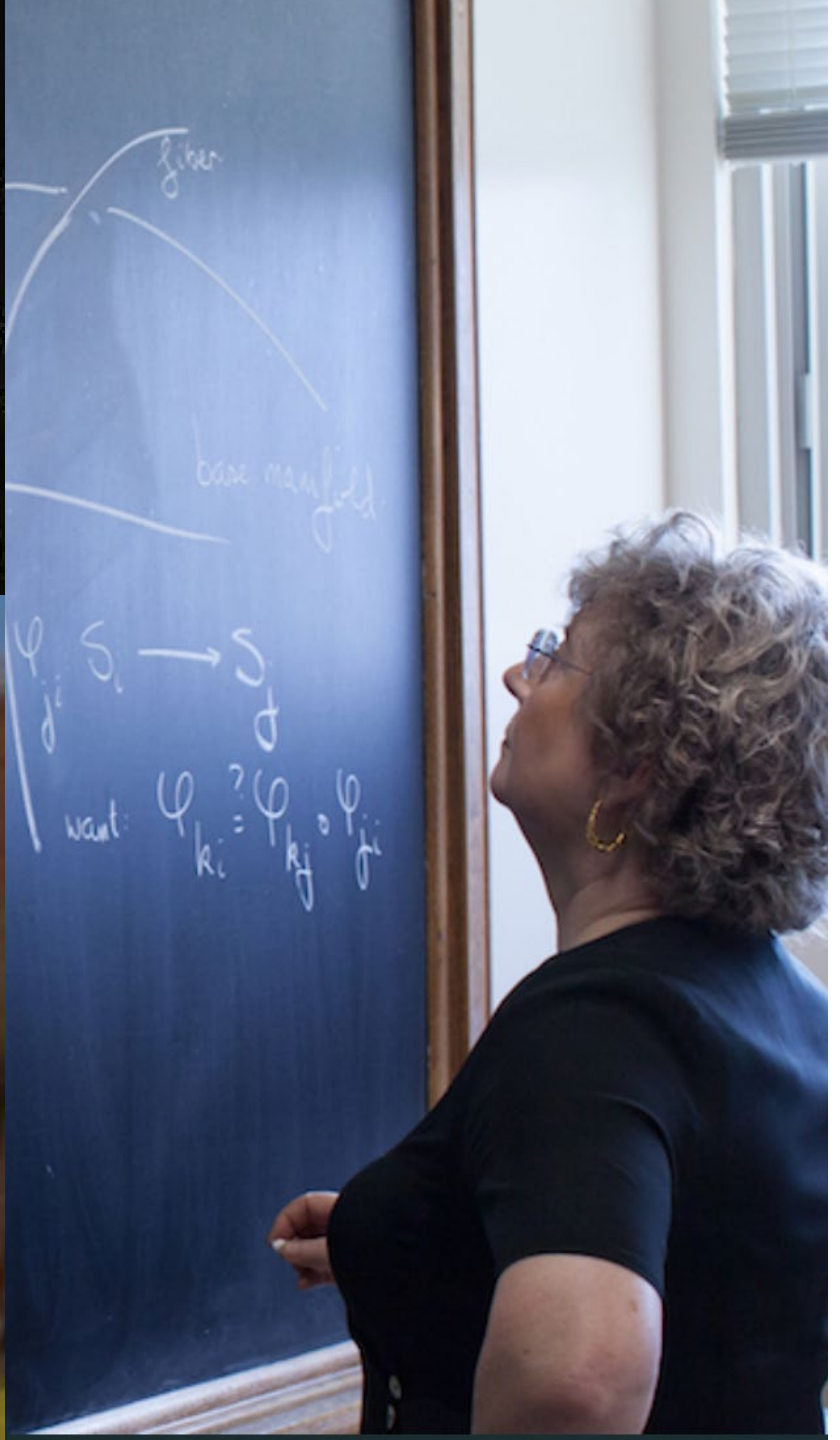
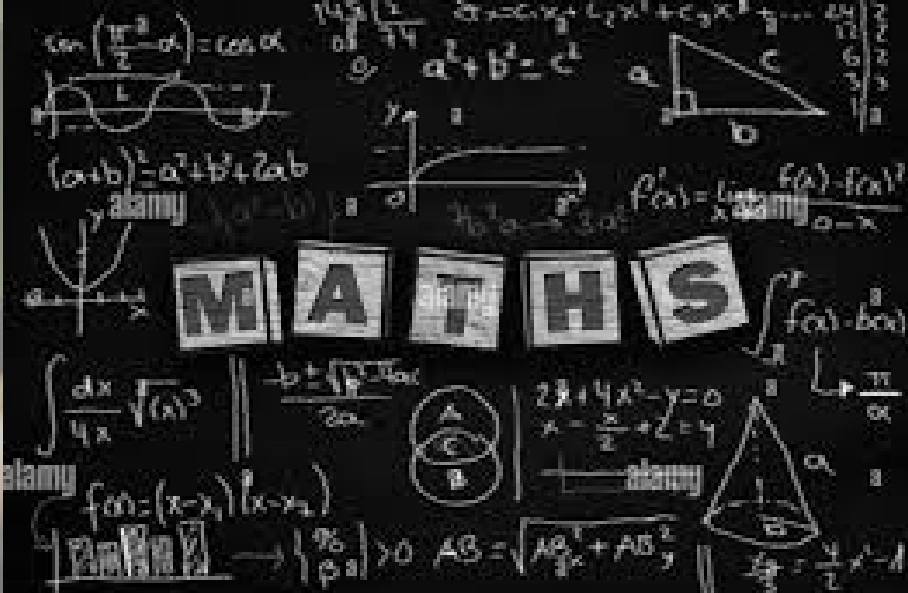
$$(X, Y) = \left(\frac{x}{1-z}, \frac{y}{1-z} \right)$$



I. Sketchnotes (visual note-taking)

II. Comics, A sequential art

(A very personal point of view!)



I. Sketchnotes (visual note-taking)

I. Sketchnotes

the 5 Basic Elements



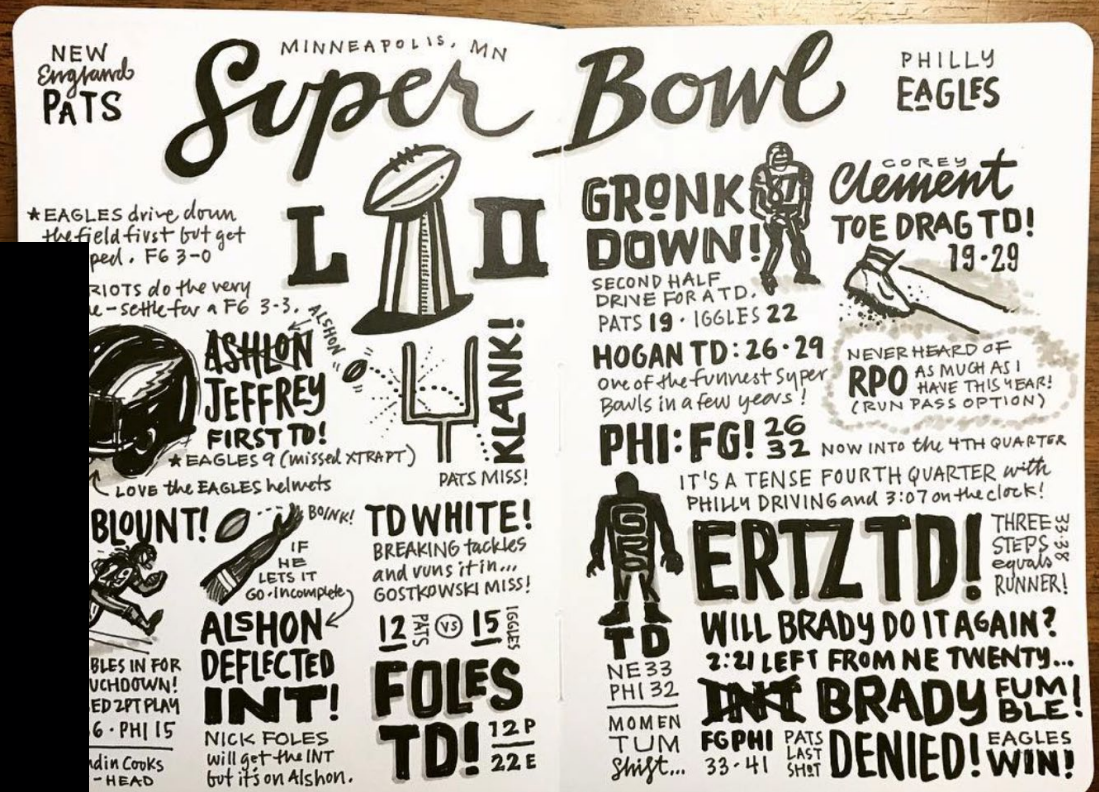
EVERYTHING YOU WANT TO DRAW CAN BE CREATED WITH THESE 5 ELEMENTS.

Can you identify the 5 basic elements in these simple drawings?

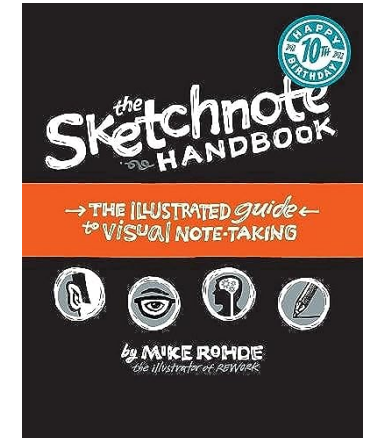


ONCE YOU REALIZE how the objects around you are made from these 5 elements, it becomes easier to draw all sorts of things.

IDEAS,
NOT
ART!



Mike Rohde,
The Sketchnote
Handbook.



#Noethember (2018)

30 days of drawings about Emmy Noether

In collab with the blog The Aperiodical, UK



The Aperiodical

Excerpt from the List of Prompts

19. Much of Noether's work in abstract algebra was studying rings – sets of objects with two different ways to combine them – such as the ring of whole numbers (integers) with addition and multiplication. Of particular interest are ideals, which are particular subsets of a ring.

22. Noether's (first) theorem states that every differentiable symmetry of the action of a physical system has a corresponding conservation law. It explains the mathematical origin of conservation of energy and momentum in physics.

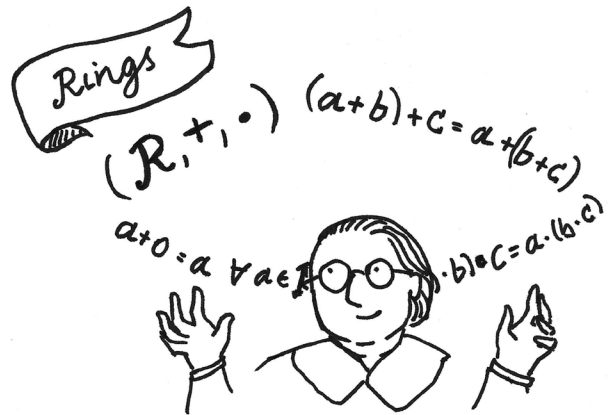
23. "If one proves the equality of two numbers A and B by showing first that $A \leq B$ and then that $B \leq A$ it is unfair; one should instead show that they are really equal by disclosing the inner ground for their equality." – Emmy Noether

Bibliography

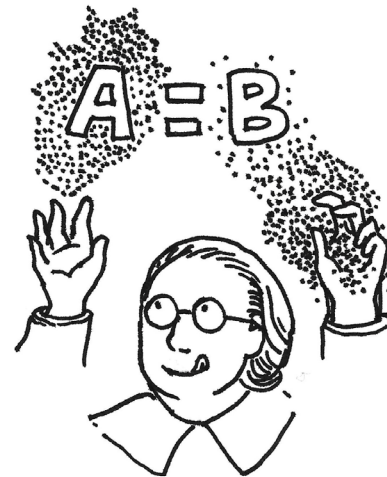
Emmy Noether in Bryn Mawr (Springer, 1983)

Emmy Noether, on Wikipedia

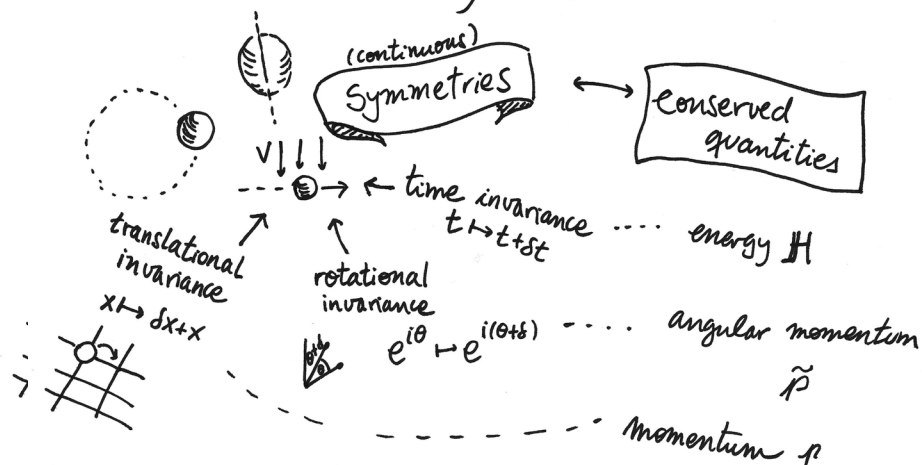
#Noethember Day 19



#Noethember Day 23



#Noethember Day 22



#Noethember (2018)

30 days of drawings about Emmy Noether

In collab with the blog The Aperiodical, UK



The Aperiodical

Search

You're reading: Events

#Noethember: a retrospective

By Katie Steckles and Constanza Rojas-Molina. Posted January 21, 2019 in Events

#Noethember Day 19

Rings

$(\mathbb{R}, +, \cdot)$ $(a+b)+c = a+(b+c)$

$a+0 = a \quad \forall a \in \mathbb{R}$ $a \cdot 1 = a$ $a \cdot (b \cdot c) = (a \cdot b) \cdot c$

#Noethember Day 23

$A = B$

#Noethember Day 22

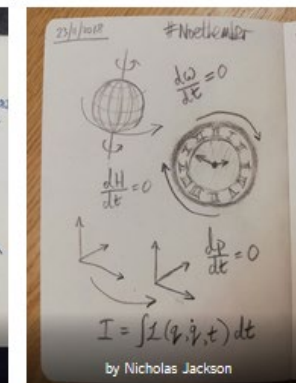
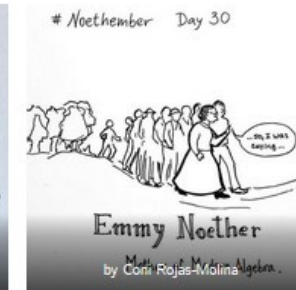
(continuous) Symmetries

conserved quantities

time invariance $t \mapsto t+st$... energy H

rotational invariance $e^{i\theta} \mapsto e^{i(\theta+\delta)}$... angular momentum \vec{p}

translational invariance $x \mapsto \delta x + x$... momentum p



#Noethember (2018)

2018

Articles published in the US (Blog of the AMS), in France (Images de mathématiques), Spain, Italy, Germany.

2021

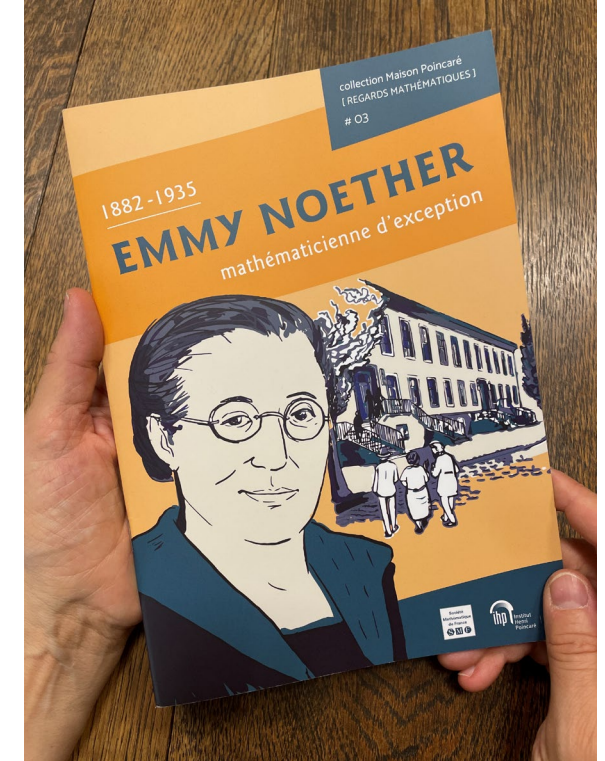
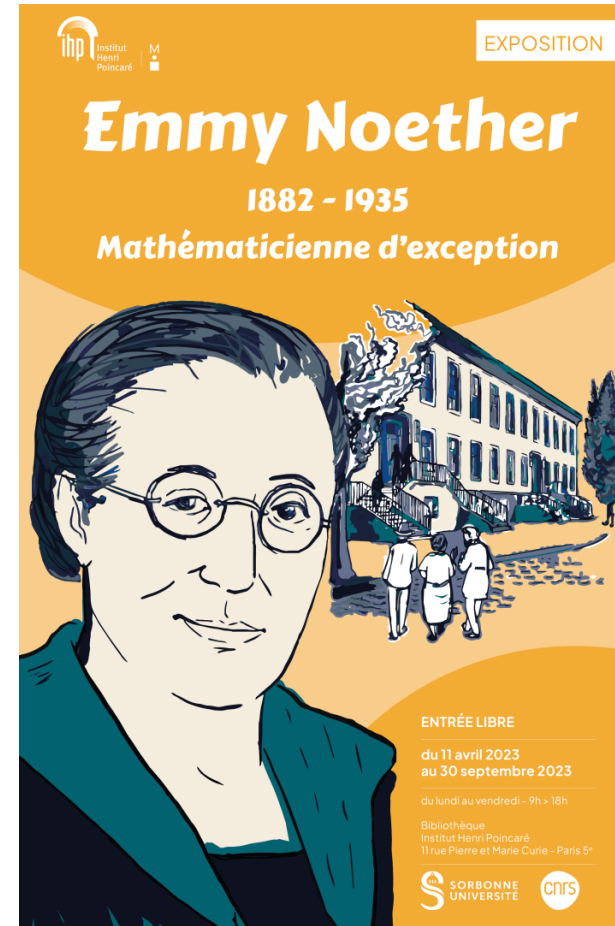
- Indian Mathematics Magazine Bhavana.

2023

- Exhibition on Emmy Noether, Institut Henri Poincaré (IHP), Paris.
- Maths/art residence Mathémartistes, U. of Angers, France.

2024

- Exhibition and reading of theater play by A. Panati (U. Toulon and Paris)



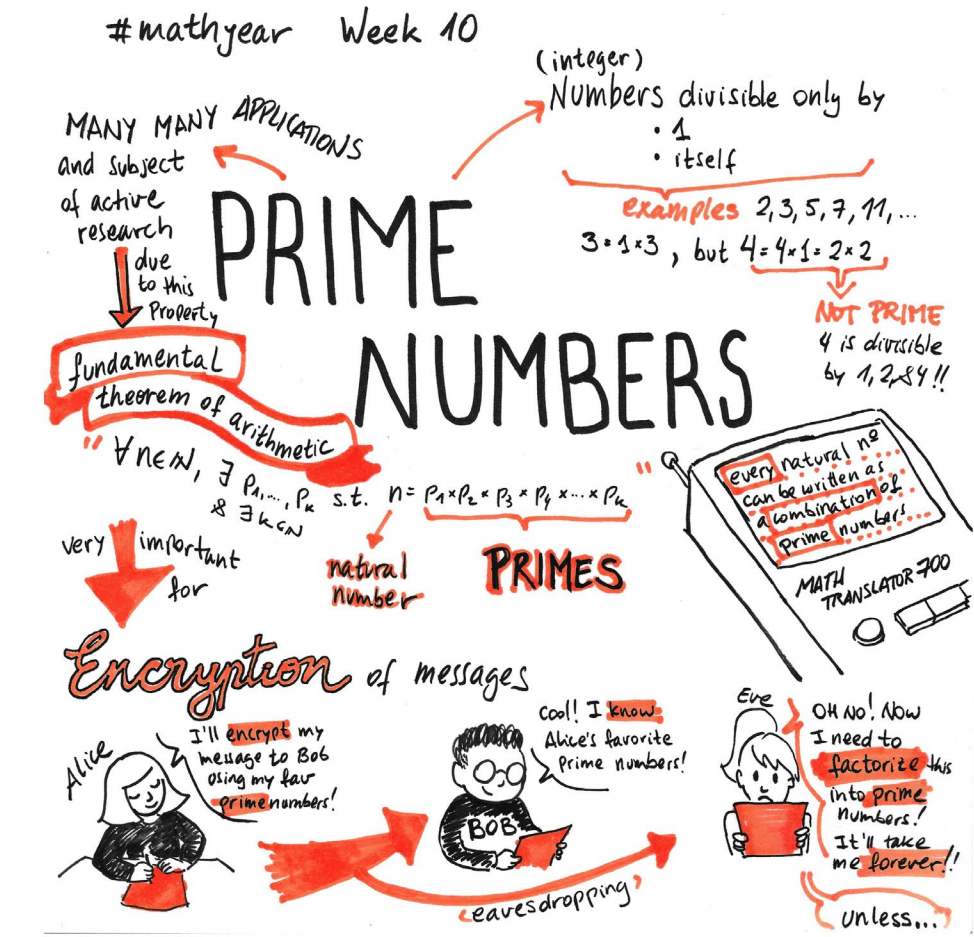
#Mathyear (2019--)

52 weeks of maths and its interactions

A drawing challenge



In collab with
Marlene Knoche
@sanguinikDE (DE)



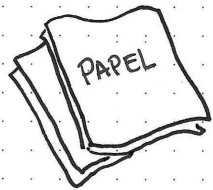
In DMV magazine

The #mathyear challenge
< User's guide >



El desafío #mathyear

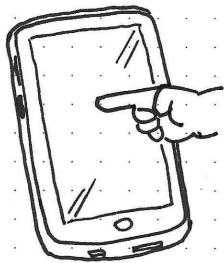
< manual de uso >



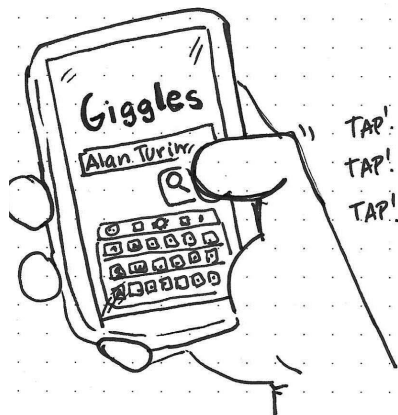
Lápices /
Crayones /
marcadores



La lista de
temas

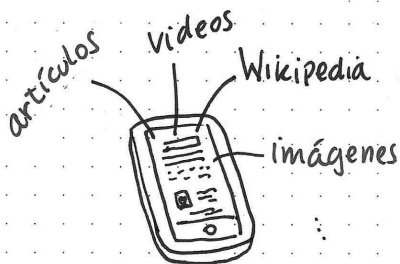


1 teléfono Smartphone
• 1 tableta
• 1 compu



TAP!
TAP!
TAP!

... y ¡a dibujar!



ah, qué lindo! ♡



15'-30' minutos



toma una
foto y súbela
a redes sociales
con el hashtag

#mathyear



Lista de temas Abril- Julio

Abril: las matemáticas como un lenguaje

- 30.03-05.04. Noah Chomsky y la jerarquía de Chomsky
- 06.04-12.04. Teoría de automatas
- 13.04-19.04. Lenguajes de programación
- 20.04-26.04. Mi fórmula favorita

Mayo: las matemáticas y la física

- 27.04-03.05. Las matemáticas como un lenguaje para la física
- 04.05-10.05. Historia de la física
- 11.05-17.05. Mi física o físico favorita
- 18.05-24.05. Las 3 leyes de Newton
- 25.05-31.05. Mecánica cuántica

Junio: el arte y las matemáticas

- 01.06-07.06. Geometría
- 08.06-14.06. La proporción dorada
- 15.06-21.06. Gödel, Escher y Bach
- 22.06-28.06. La música



#mathyear Week 31

SPACE TIME

<one word>

a very special world where every particle is described by 4 coordinates

SPACE (x, y, z, t)

TIME $t=0, t=1, t=2, t=3, \dots, t$

in this space, Gravity is the **CURVATURE** of the space

PAST PRESENT FUTURE

Minkowski Space (1908)

Hermann Minkowski

#mathyear Week 8

the mathematics of Behavior

GAME THEORY

ARTIFICIAL INTELLIGENCE learning & memory

PROBABILITY **RANDOM GRAPHS** Instagram Followers

STATISTICS PSYCHOLOGICAL TESTS A B

NON-LINEAR DYNAMICS interacting systems Predator & prey

SOCIAL DYNAMICS

BIG DATA HMM

Source: E. Hunt. "The Mathematics of Behavior"

#mathyear Week 32

In early 1900s: Genetics $\xrightarrow{\text{connection}}$ Evolution

BIOSTATISTICS

Statistical methods applied to **BIOLOGY**

TODAY: guidelines for **EXPERIMENT DESIGN**

start with a question **Q** state a hypothesis **H**

Collect **DATA**

Sample from population

STATISTICAL ANALYSIS

INTERPRETATION AND CONCLUSION does the data validate H?

APPLICATIONS: PUBLIC HEALTH, CLINICAL TRIALS FOR MEDICINE, EPIDEMIOLOGY

it's all over the NEWS!

#mathyear Week 10

PRIME NUMBERS

MANY MANY APPLICATIONS and subject of active research due to this Property

fundamental theorem of arithmetic

very important for **Encryption** of messages

(integer) Numbers divisible only by 1 & itself

examples 2, 3, 5, 7, 11, ...

$3 = 1 \cdot 3$, but $4 = 1 \cdot 2 \cdot 2$

NOT PRIME 4 is divisible by 1, 2 & 4!!

natural number **PRIMES**

s.t. $N = P_1 \cdot P_2 \cdot P_3 \cdot P_4 \cdot \dots \cdot P_n$

every natural no can be written as a combination of prime numbers!

MATH TRANSLATOR 700

Alice: "I'll encrypt my message to Bob using my favorite prime numbers!"

Bob: "Cool! I know Alice's favorite prime numbers! It's nice to have an eavesdropper!"

Eve: "OH NO! Now I need to factorize this into prime numbers! It's nice to have an eavesdropper! unless..."

#mathyear Week 13

the enigma machine

electro-mechanical device that uses rotors to encrypt msgs

used in **WWII**

in 1939, Poland shared its findings with France and UK

Marian Rejewski Polish mathematician

led efforts in Poland to decipher the functioning of

in Bletchley Park Alan M. Turing led the efforts to decipher the Enigma code using an improved bombe

bombe invented to decipher Enigma

rotors, keyboard (input), lamp board (output), adds complexity to cipher, extra level of complexity

complicated steps to go from A to D + many possible configurations = encrypted message very hard to decipher!

all their efforts combined helped the Allies win WWII

#mathyear Week 28

there are different kinds of infinity:

INFINITY

is not a number it's an idea

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6

1, 1.2, 1.5, 1.7, 2, 2.2, 2.5, 2.7, 3, 3.2, 3.5

1, 1.24, 1.25, 1.5, 1.55, 1.7, 1.74, 1.75, 2, 2.22

1, 1.1, 1.15, 1.21, 1.27, 1.3, 1.37, 1.4

can't count this!

can't count this!

INFINITY AND BEYOND!

Our universe is finite in extension but, oh so large for us

OUR UNIVERSE $\approx \infty$

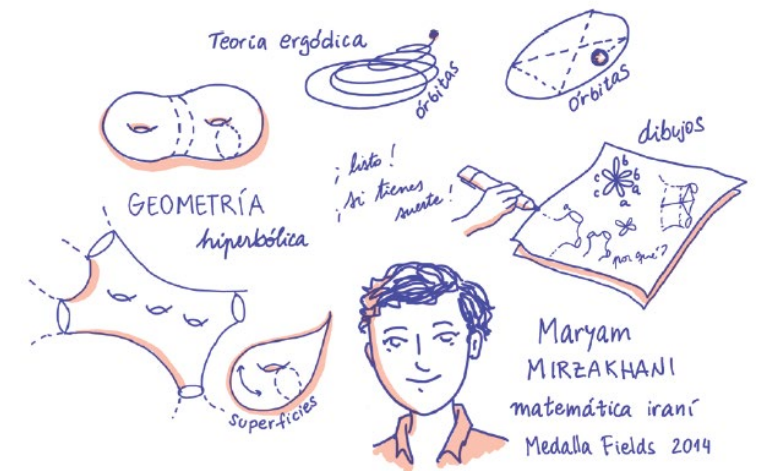
GRANNIA I LOVE YOU INFINITY!

More info in The Handbook of Mathematical Science Communication (A.M. Hartkopf/ E. Henning)

La gran aventura, 2023



Leslie Jiménez Palma (U. Chile)



In the classroom

Bachelor in Data Science,
CY Cergy Paris Université - 2020

COMPUTER SCIENCE

python: algorithmics and programming

DATA TYPES

int: "hello", "goodbye"

float: 2.5, False

string: "hello", "goodbye"

boolean: False, TRUE

control structures

CONDITIONS: IF... ELSE...

loops: WHILE [Condition], FOR [range]

OPERATORS

* // **

/ += mod

BREAK

OBJECTS

STACKS: 3, 2, 1, 0

MATRICES: 10 1 8, 2 4 9, 3 8 9

QUEUES: 0, 1, 2, 3

ARRAYS: 1, 0, 4, 9, 10

TUPLE: 0, 1, 2, 3

libraries

NUMPY: zeros(3,3), ones(3,3)

MATH: e, sqrt, random

BUILT-IN: MAX(), MIN(), A(n)

FUNCTIONS

recursive

```
fact(n):
  if n = 1,
  | return(n)
  Else
  | return(n * fact(n-1))
```

non recursive

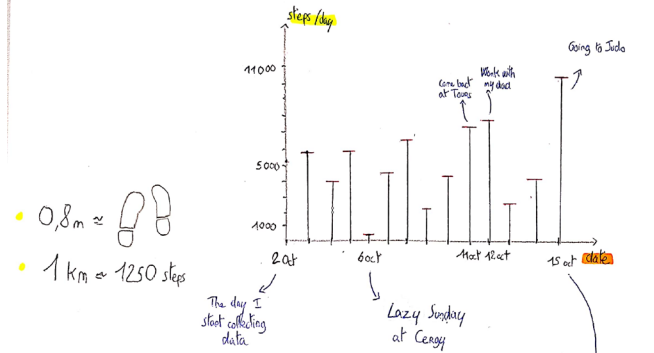
```
fact(n):
  r = 1
  for i in range(2, n+1)
  | r = r * i
  return r
```

sorting

BUBBLE SORT

FACTORIAL!

How is my daily walking ?



- 0,8m ≈ 1000 steps
- 1 km ≈ 1250 steps

Kcal

6 Oct: * * * * *

15 Oct: * * * * *

The RECORD!!!

The place where I broke the record

Bois de l'oise (Cergy)

Inspired by Dear Data,
by Lupi and Posavec



Next goals (w. L. Jiménez):

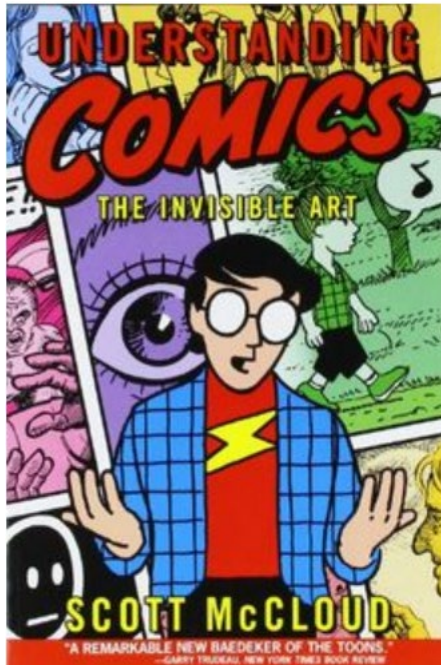
- Create didactic material for teachers based on book La gran aventura (CH and FR)
- Design learning experiences
- Evaluate impact

II. Comics, a sequential art

II. Comics

Examples of comics about maths

- The Cartoon Guide to... Gonnick, 1993
- Logicomix, Doxiadis, Papadimitriou, 2008
- Emmy Noether (online comic), Agrapidis, Mistrello, 2019
- Infinix, Schafer, 2021
- Sofia Kovalevskaya, Milani, 2023

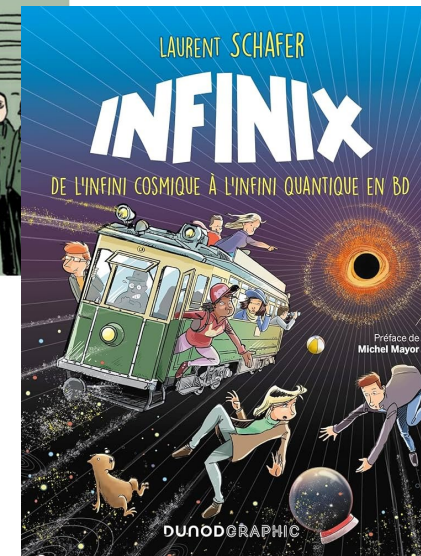
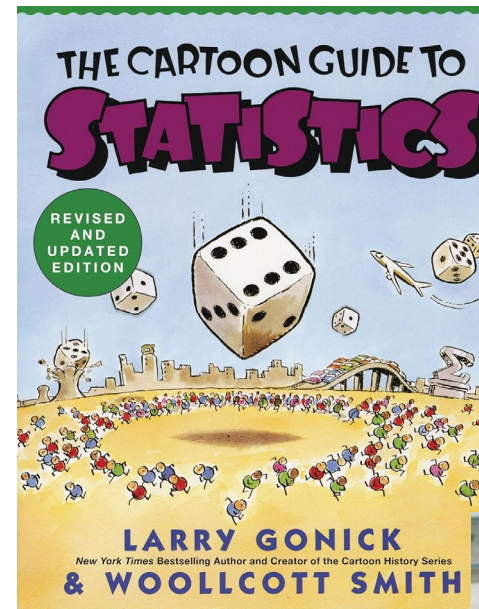


EMMY NOETHER CLIÒ AGRAPIDIS
ELENA MISTRELLO



About comic as format

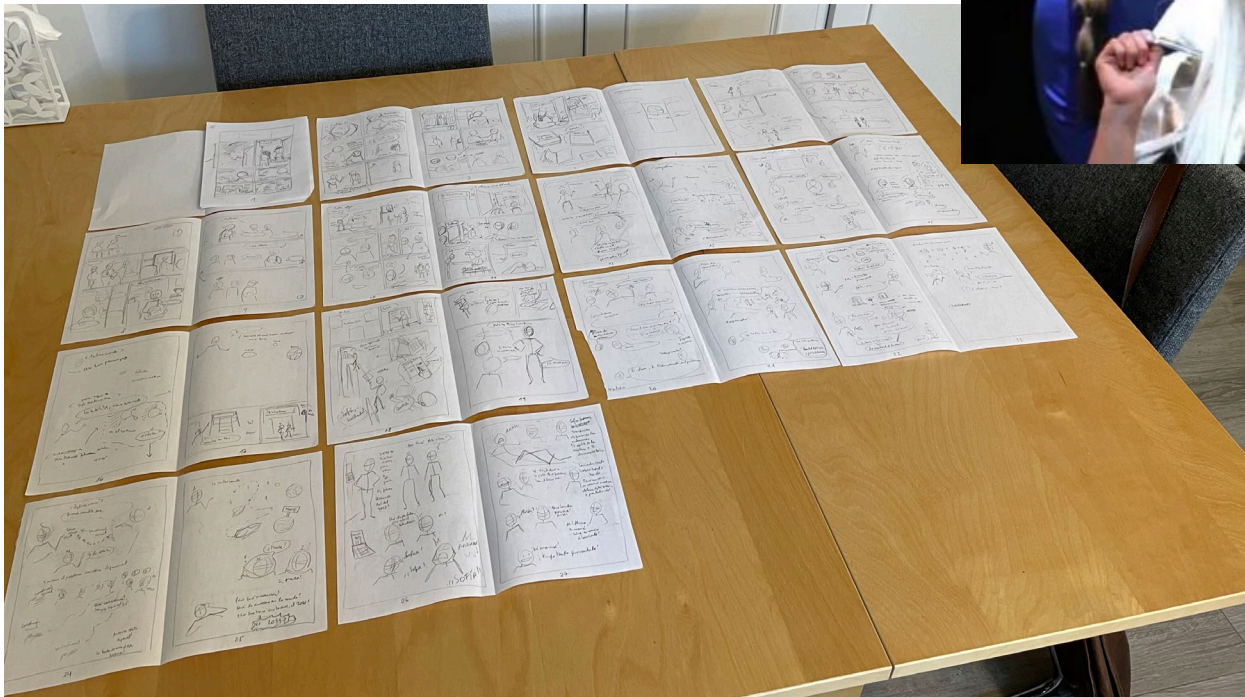
- Understanding Comics, McCloud, 1993
- *Unflattening*, Sousanis, 2015
- M. Farinella, The potential of comics in science communication, Journal of science communication, 2018



Comic "Alicia's mysterious conjecture"



Comic adaptation of the theatre play by mathematician Alberto Mercado (U. T. FSM, Chile)



Play « 4,2,1 »
Theater troupe
La coraje,
Valparaíso, Chile



The Collatz Conjecture



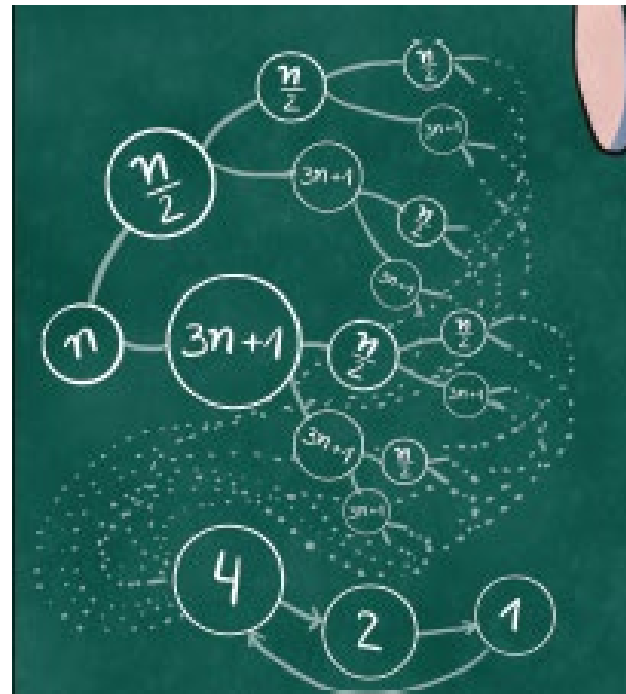
Scientific content

- The Collatz Conjecture or $3n-1$ problem
- Theorem and conjecture
- Biographies of scientists...



Lothar Collatz

(1910-1990) MATHÉMATICIEN ALLEMAND. IL ÉTAIT CONVAINCU DE LA RESPONSABILITÉ DES MATHÉMATIENS D'UTILISER LEURS CONNAISSANCES POUR RÉSOUDRE LES PROBLÈMES DU MONDE. IL A ÉNONCÉ LA CONJECTURE EN 1957.



... ET TOUTS LES MEMBRES D'UNE ÉQUIPE NE SONT, POUR SÛR, PAS TOUJOURS RECONNUS




Rosalind Franklin

(1920-1958) CHIMISTE BRITANNIQUE. APRÈS AVOIR OBTENU SON DOCTORAT, ELLE A TRAVAILLÉ ET DÉVELOPPÉ UNE VASTE CARRIÈRE DE CRISTALLOGRAPHIE À RAYONS X. ELLE A ÉTÉ PIONNIÈRE DANS LA DÉCOUVERTE DE LA STRUCTURE MOLÉCULAIRE DE L'ADN, UN RÉSULTAT POUR LEQUEL ELLE N'A REÇU AUCUNE RECONNAISSANCE.

Katherine Johnson

(1918-2020) MATHÉMATICIENNE AMÉRICAINE. ELLE A OBTENU SON DIPLÔME DE MATHÉMATICIENNE À UNE ÉPOQUE DE SÉGRÉGATION RACIALE AUX ÉTATS-UNIS, OÙ LES PERSONNES NOIRES N'ÉTAIENT GÉNÉRALEMENT QUE JUSQU'À L'ÂGE DE 14 ANS. ELLE A TRAVAILLÉ COMME INSTITUTRICE, A ENTAMÉ DES ÉTUDES SUPÉRIEURES EN MATHÉMATIQUES ET A MENÉ UNE LONGUE CARRIÈRE DE CALCULATRICE À LA NASA, PARTICIPANT À D'INOMBRABLES MISSIONS SPATIALES.



CONJECTURE **THÉORÈME**

UNE CONJECTURE EST UNE AFFIRMATION QUI SEMBLE VRAIE, MAIS DONT ON N'EST PAS TOUT À FAIT SÛR.

À LA DIFFÉRENCE D'UN THÉORÈME...

DONT ON SAIT QU'IL EST VRAI.

COMME... LE THÉORÈME DE PYTHAGORE ?

... EXACTEMENT ! LA PREUVE DE CE THÉORÈME EST BIEN CONNUE.

EN FAIT, IL EXISTE PLUSIEURS PREUVES !

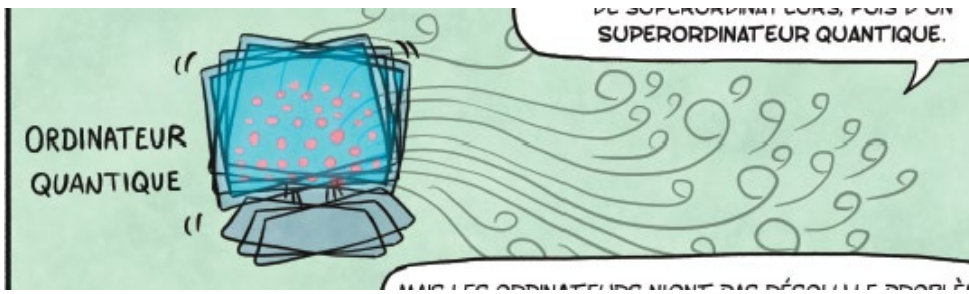
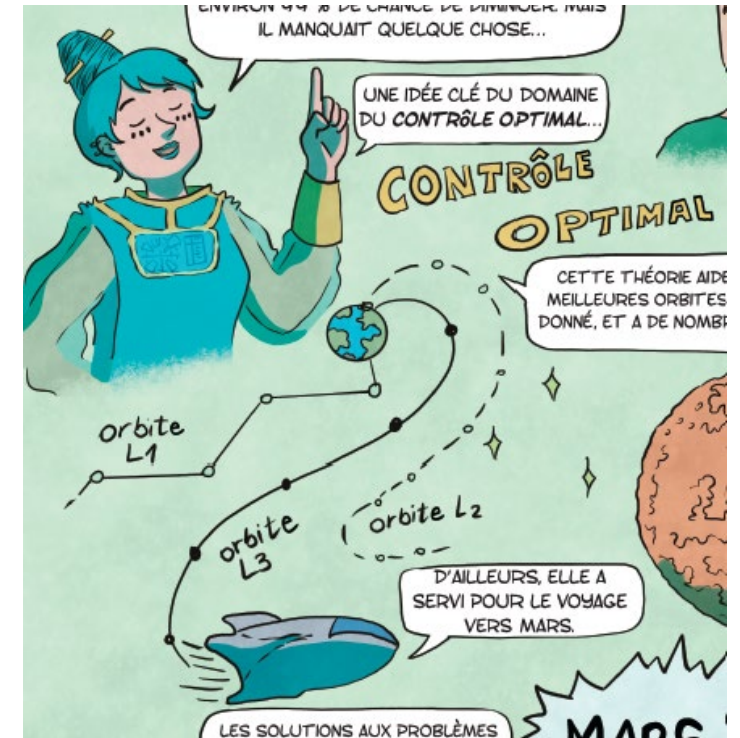
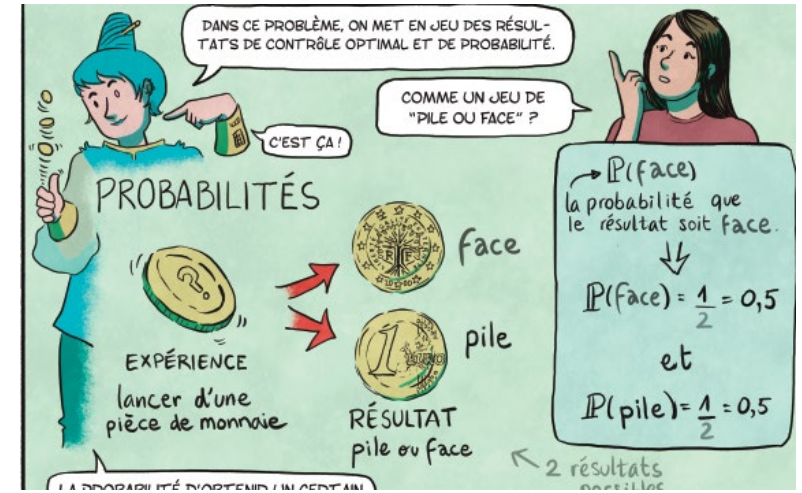
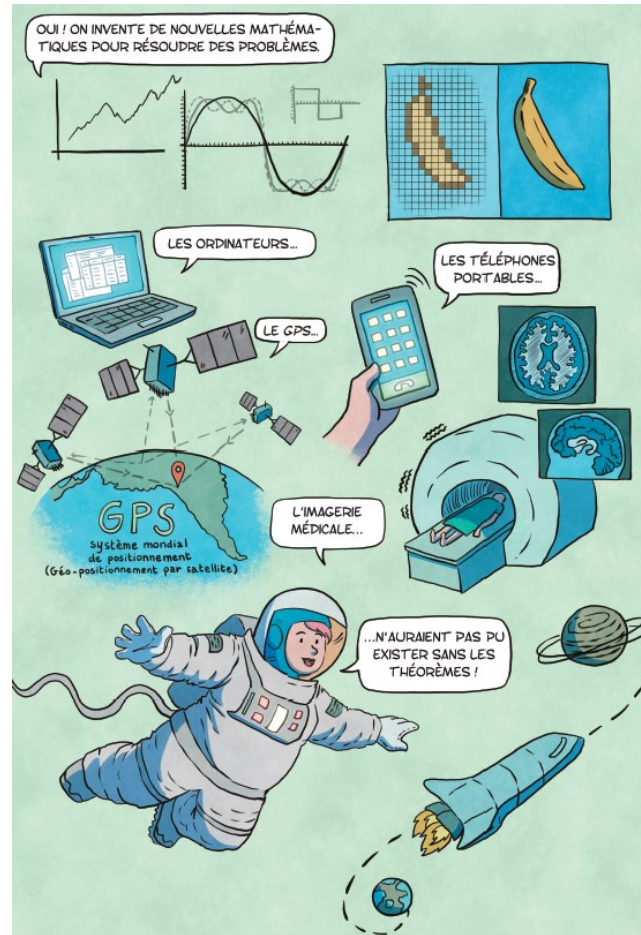
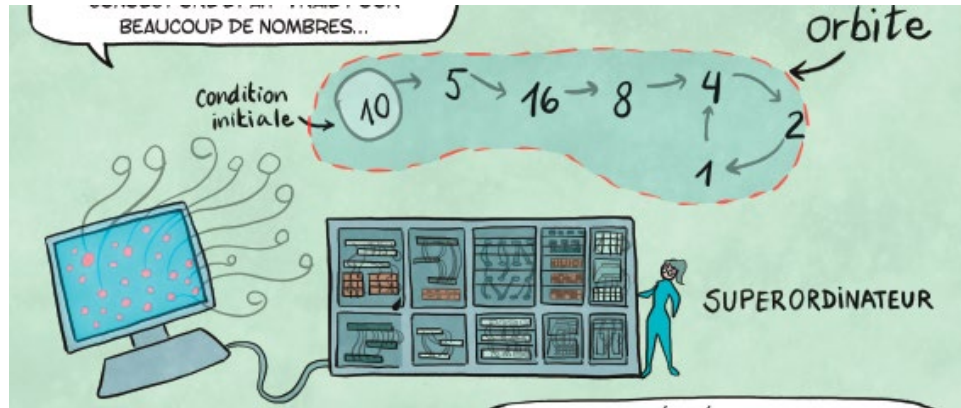
OUI ! NOUS LES AVONS VUES EN CLASSE, TU TE SOUVIENS, NICOLÁS ?

EHH...

ce symbole indique la fin de la démonstration

Scientific content

- Fields of research in mathematics
- Scientific advances in which Mathematics have several applications and might take time



The learner's ecosystem



Impact evaluation

High schools in Bayeux, France

Lycée Alain Chartier (Prof. Olivier Longuet)

Lycée Jean d'Arc (Prof. Flavie Auburg)

Seconde – Première - Terminale

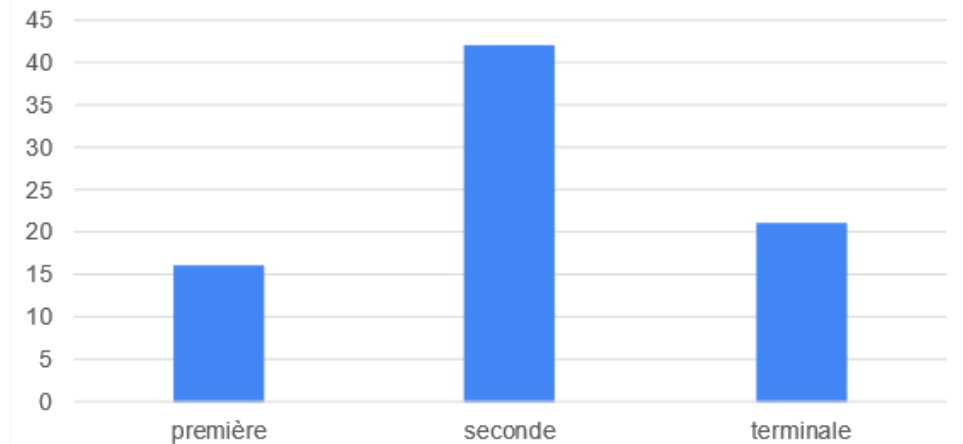
105 answers PRE reading, 79 answers POST reading

Troisième 15 PRE, 11 POST

Positive: characters are the same age as readers, interactions with family and friends, clear language and clear drawings, biographies

To improve: definition of word « orbit », more details in mathematical notions, balance text/images.

Teacher's remarks: rich in ideas, personal development, role of mistakes, good ideas for « grand oral » and problems to work in class. Esthetics better suited for a younger audience

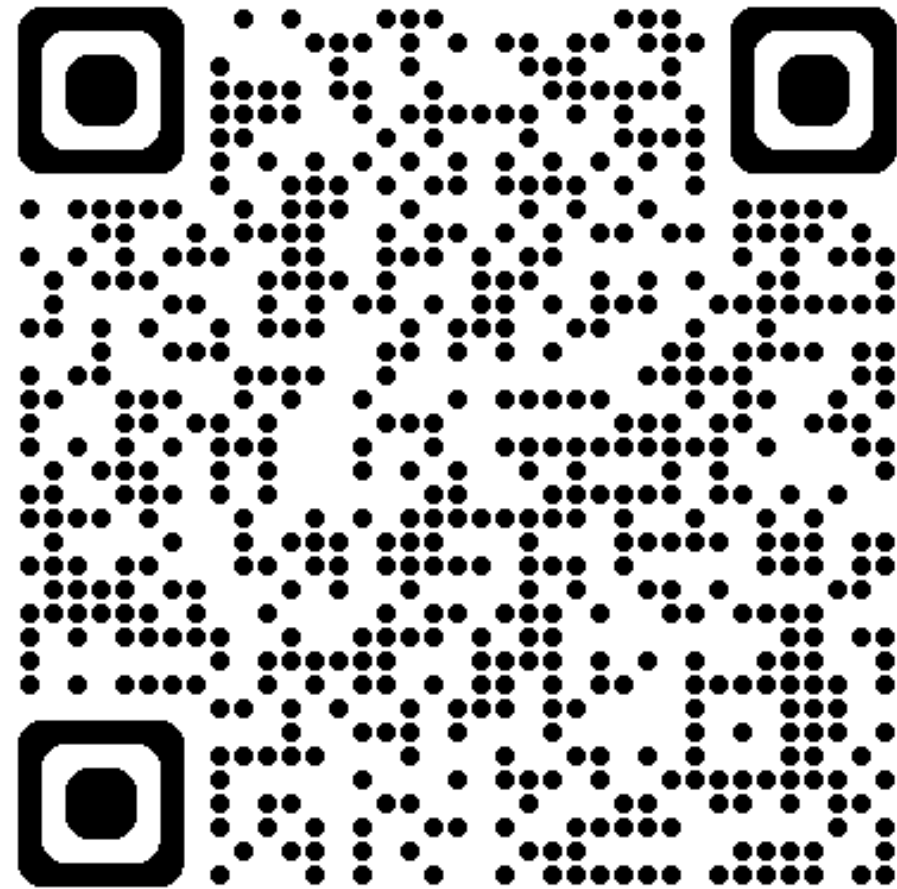


From « Mathematics communication through comics: « Alicia's mysterious conjecture''', by C.RM, Proceedings of the Conference Telling Science, Drawing Science, Angoulême, November 2024.

Knowledge acquisition: The post-reading survey asks students to select the statement that corresponds to the Collatz conjecture and to indicate if it is the work of one person or of many. Among lycée students, a vast majority (88.8%) selects the correct statement of the Collatz Conjecture and the fact that several people worked on it. Among collège students, 63.6% selects the correct statement of the Collatz Conjecture, while 90.0% select correctly the fact that several people worked on it.

Comparing concepts before and after reading the comic: The pre- and post-reading surveys ask the students to select a statement that describes a theorem, and one that describes a conjecture. In the pre-reading survey among lycée students, a vast majority (83%) indicates correctly that a theorem is a statement that has a proof (that is true because it can be deduced from other known principles). In the post-reading survey this percentage rises to 91,3%. While in the pre-reading survey only 47,2% of lycée students indicate correctly that a conjecture is a statement that is expected to be true but whose proof is not known, this percentage rises to 73,8% in the post-reading survey. In the pre-reading survey, 29,9% of lycée students believe a conjecture is a statement that is partially proven, which goes down to 16,3% in the post-reading survey. Among collège students the situation is similar: before reading the comic, 73,3% of students select the correct description of theorem, which goes up to 81,8% after reading the comic. Before reading the comic, 26,7% select the correct description of conjecture, which goes up to 54,5% after reading the comic. Before reading the comic, 53,3% of students think a conjecture is a statement that is partially proven, which goes down to 36,4% in the post-reading survey.

We conclude that the comic has been effective in explaining the difference between conjecture and theorem and in explaining the Collatz conjecture.



www.crojasmolina.com/comic-acip

Sketchnotes and comics

- Visual languages
- Means of communication, not (necessarily) art.
- Cheap format – massive format
- Different time scales: sketchnotes can be produced very quickly, comics need time (fanzines are a short and quick format *).
- Sketchnotes needs context, so it needs to be accompanied by text or a person. Comics provides the context.
- Space for subjectiveness and personal expression → Achtung!
Needs reviewing, as any other science communication format.
- Potential as didactic material.

Outlook:

Team up with didacticians of mathematics to design learning experiences to evaluate the instrument (comic/sketchnote) improve it, evaluate its impact and its scope.

Potential as didactic material

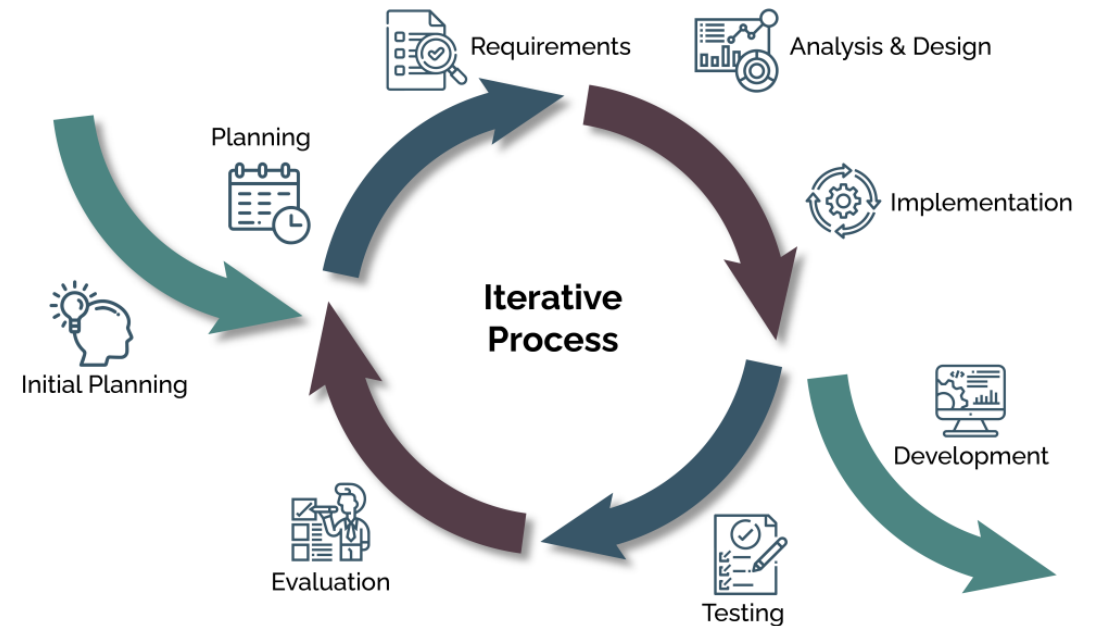
Instrument for science communication with potential for science education.

- Prepare didactic material to use in the classroom
- Need of (comparative) understanding of curricula and educational approaches in (different) countries.
- Problem: math research is **international**,
math education isn't!

A design approach for future projects:

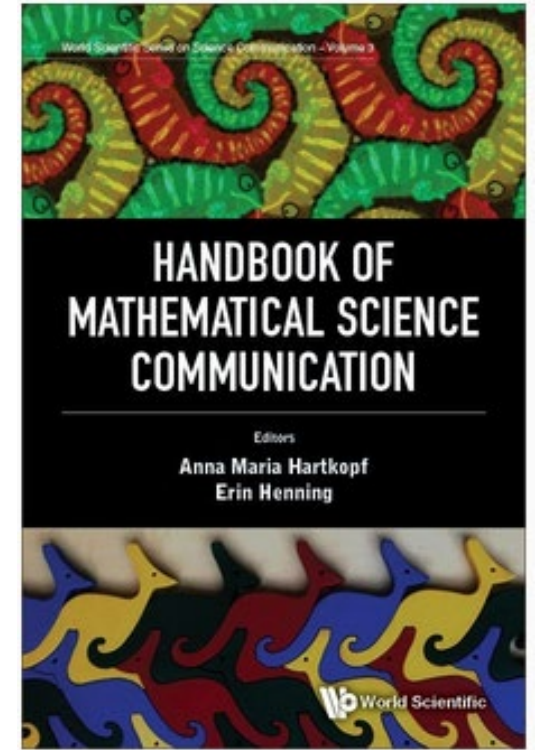
High cost: time is a resource, feedback is precious!

Iterative Process Model



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