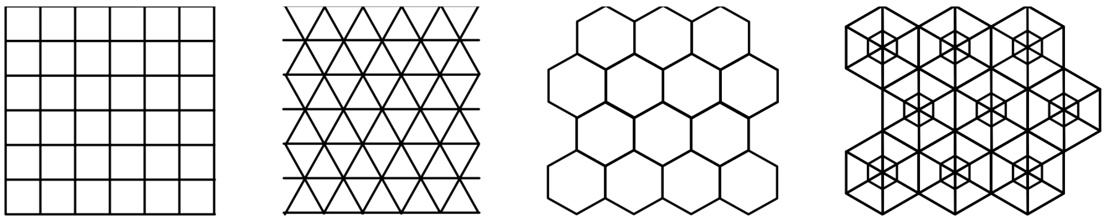
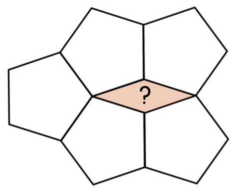
SPECTRES, HATS AND MATHS

Mathematicians studied since a long time what shapes can cover an entire plane, an infinite floor, without gaps or overlaps. This is easy if you can use any shape you like, but can be very hard if you restrict yourself to a small set of shapes. Even harder with just one!

An easy solution, is to cover the floor with figures that repeat themselves over and over in a regular way. You can do it with squares, equilateral triangles and regular exhagons for example.



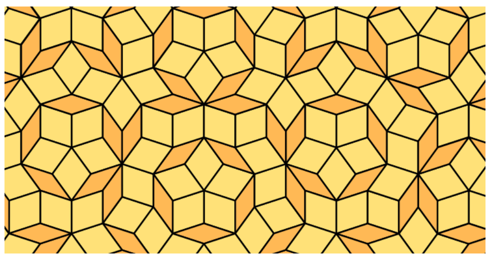
But already a pentagon will cause you troubles. No matter how hard you try, you'll end up with little gaps that just won't fit another pentagon!



All these patterns look very regular, they have symmetries: you can move them in certain ways, by sliding or rotating the figures around, and they will overlap again and again, looking completely identical.

Can one arrange the tiles so that there's no symmetry, so that wherever we look in the plane, we will see something different? And, if yes, can we find one shape that can only cover the plane in this strange way? That would be what we call an aperiodic tiling.

The answer is yes: in 1974 mathematician and Nobel laureate Roger Penrose found a pair of shapes - a kite and a dart - that together tile the plane, but never with a symmetry.



Only last year, D. Smith, J.S. Myers, C.S. Kaplan, and C. Goodman-Strauss have found a single shape to do the same: the hat. Unfortunately, this needed to be flipped every now and then to tile the plane, so the quest was not over yet!

They did not give up and, just a week later, they found the spectre: this one tiles the plane aperiodically, but make sure you don't flip it!

Today we will use the hat and the spectre tiles to build our own aperiodic wall tiling!

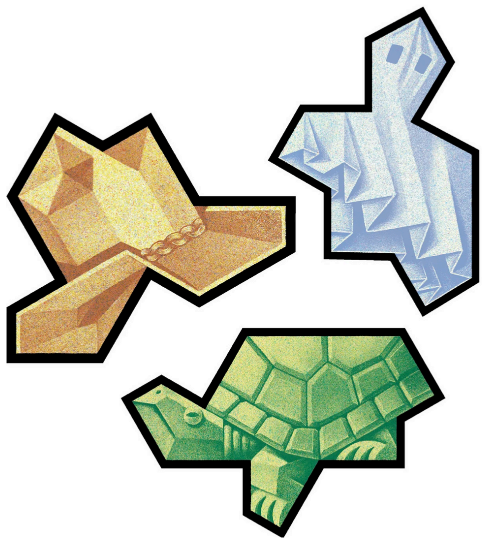


Image: C.S. Kaplan, The Missing Piece: The quest for an einstein tile - a shape never seen before in mathematics, Scientific American, Volume 330 (January 2024)