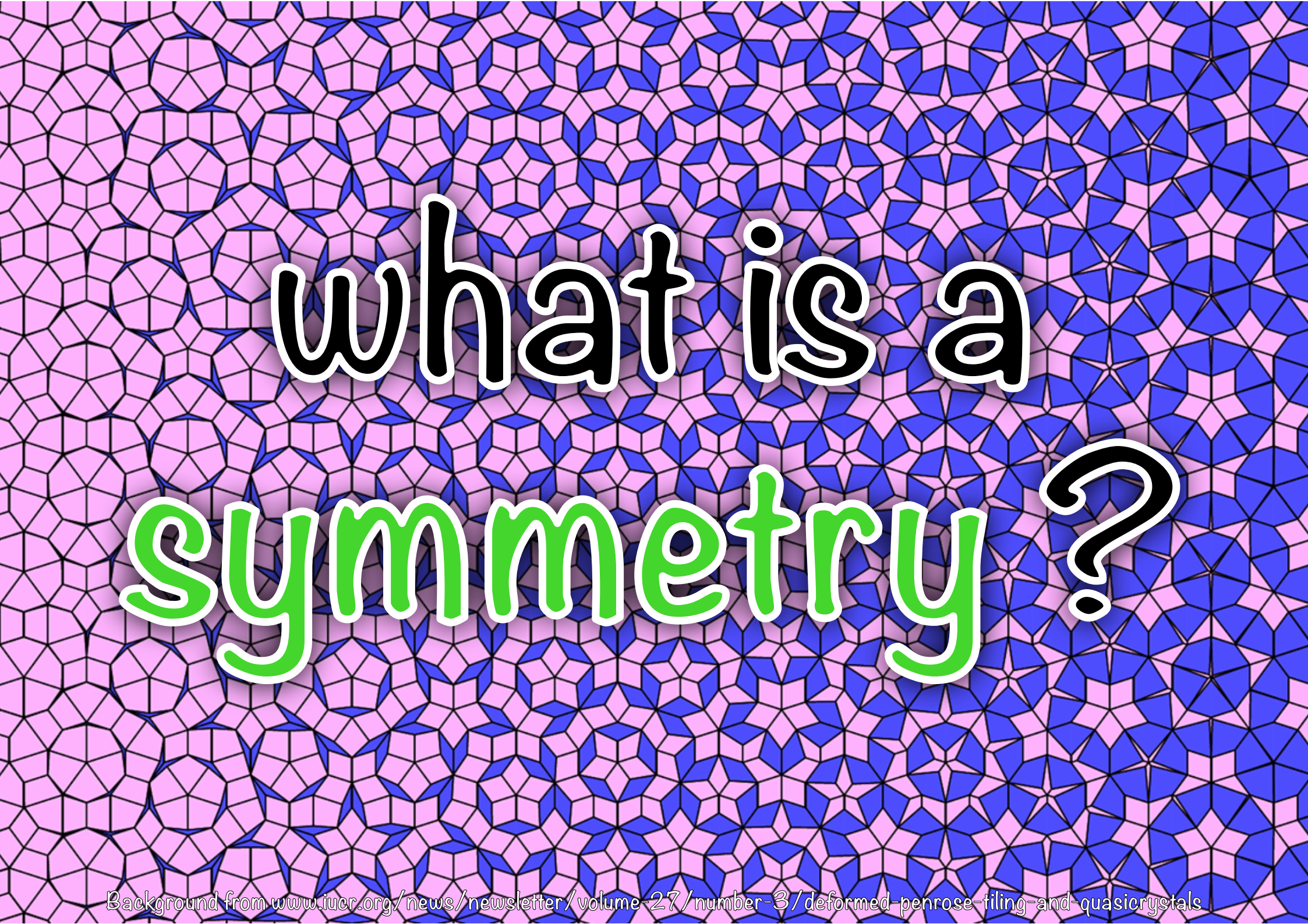




what is a symmetry?

Symmetry is when something looks the same after you move, flip, or turn it.



mirror
symmetry



translation
symmetry



rotation
symmetry

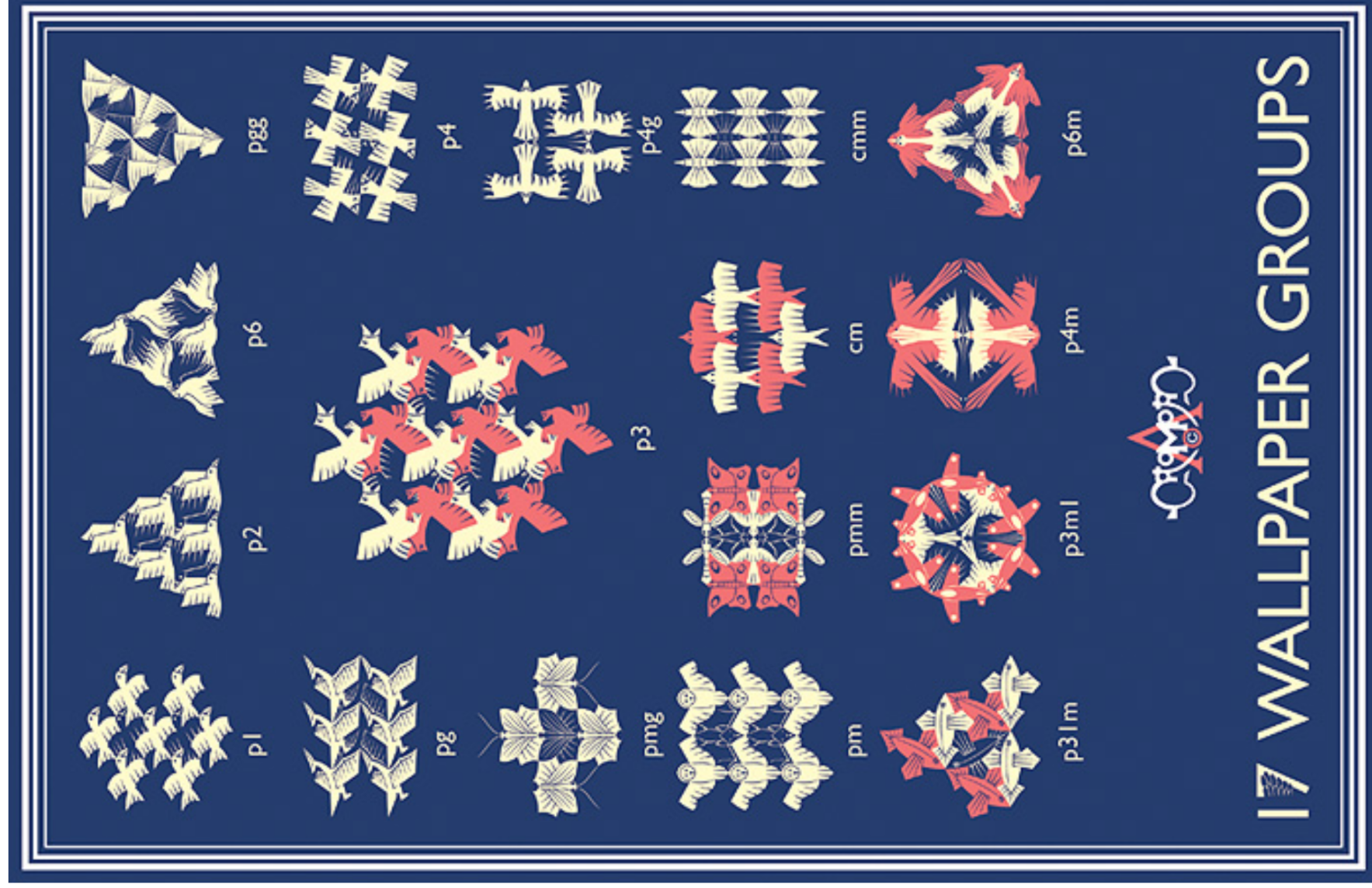
wallpaper symmetries

Imagine you're designing the wallpaper for your room.

The pattern should repeat perfectly in all directions: no gaps, no overlaps.

How many different patterns can you make?

Only 17!



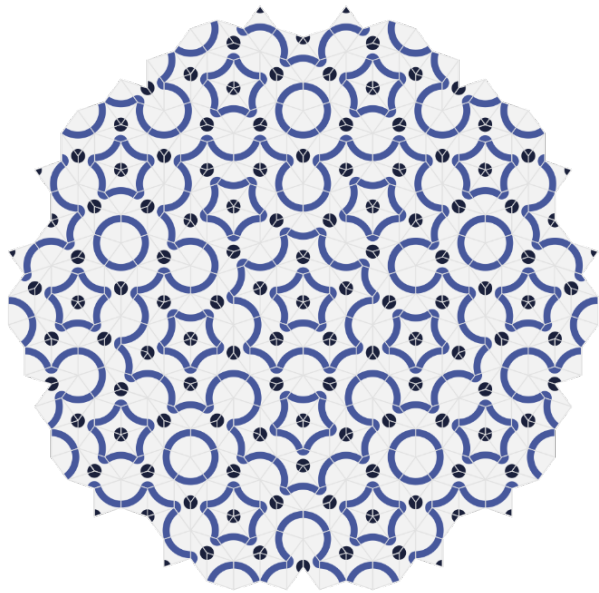
We just learned that there are only 17 ways to repeat a pattern perfectly in two dimensions.

These belong to the **wallpaper group**, and are some sort of secret rules for making patterns that repeat forever without gaps.

But what if I told you there is a way to cover the wall **where the pattern never repeats?**

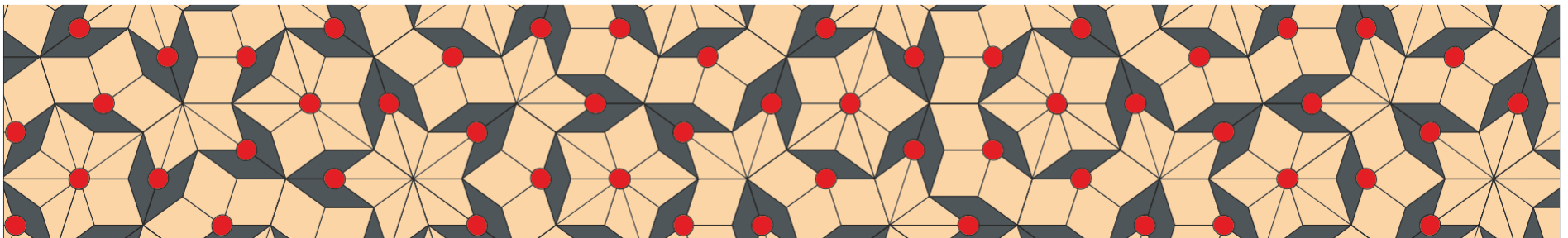
No matter how far you go, it always looks new!

These are called **aperiodic tilings**. They are like the rebels of the symmetry world!



Instead of repeating the same thing over and over, they use special shapes that fit together in a way that never makes the exact same picture twice.

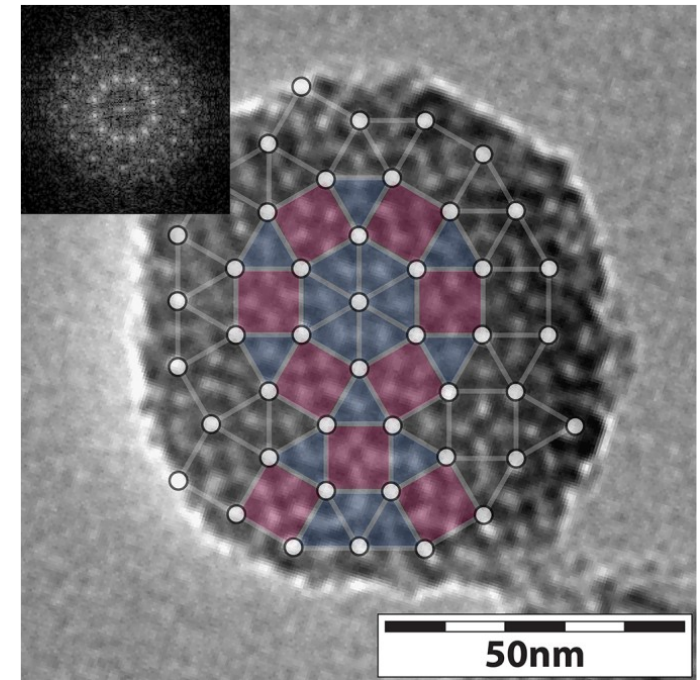
Like a puzzle that never ends!



Figures from en.wikipedia.org/wiki/Aperiodic_tiling and aperiodical.com/2023/03/an-aperiodic-monotile-exists

Some crystals and even the arrangement of atoms in certain materials use aperiodic patterns. It's not just cool maths, it's science too!

Figure from news.cornell.edu/stories/2017/08/wiesner-team-images-tiny-quasicrystals-they-form



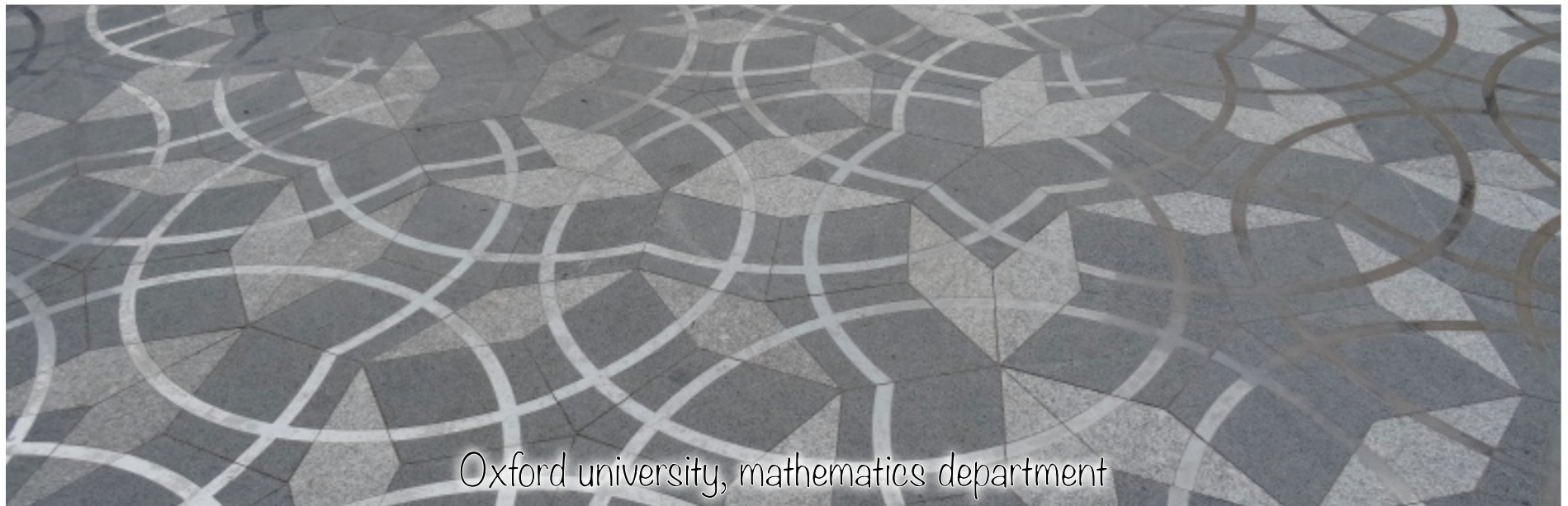
And that is not all. Artists and architects use aperiodic tilings to design things that look interesting and surprising.

P. Bourke.- The Federation Square buildings in Melbourne, Australia
Photo from mathworld.wolfram.com/AperiodicTiling.html

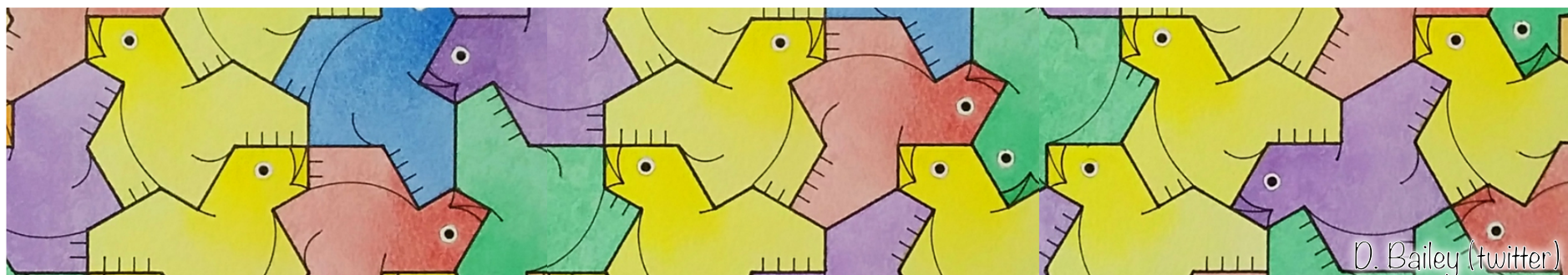
The most famous pattern was discovered in 1974 by the mathematician and Nobel laureate **Roger Penrose**.

He found a pair of shapes, a **kite** and a **dart**, that together tile the plane, but never with a symmetry.

We already encountered them in the previous pages and on the cover of this booklet!

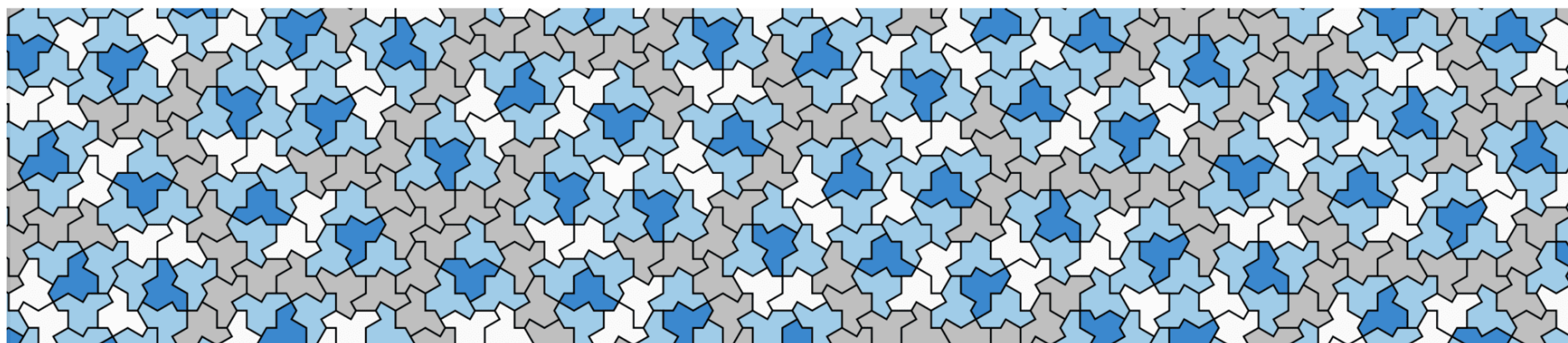


Oxford university, mathematics department

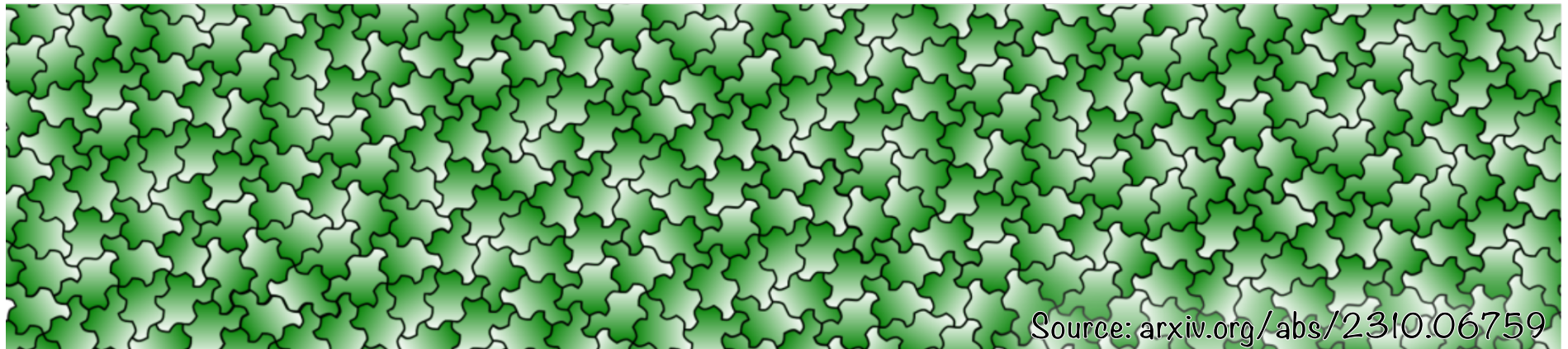


Only in 2023, D. Smith, J.S. Myers, C.S. Kaplan, and C. Goodman-Strauss have found a single shape to do the same: the **hat**.

Unfortunately, this needed to be flipped every now and then to tile the plane, so the quest was not over yet!



They did not give up and, just a week later, they found the **spectre**: this one tiles the plane aperiodically even if one does not flip it!



Source: arxiv.org/abs/2310.06759

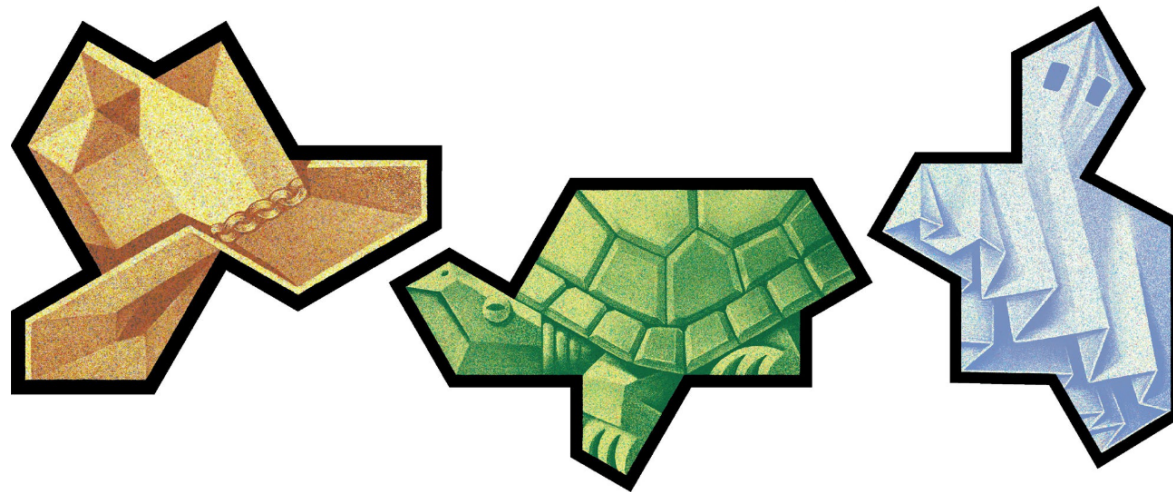


Figure by Miriam Martincic, www.scientificamerican.com/article/inside-mathematicians-search-for-the-mysterious-einstein-tile



In 2024 we combined 657 tiles, can we beat our record?